1. Active medium. The active medium consists of a collection of atoms, molecules, or ions (in solid, liquid, or gaseous form) which is capable of amplifying light waves. Under normal circumstances, there are always a larger number of atoms in the lower energy state than in the excited energy state. An electromagnetic wave passing through such a collection of atoms is attenuated; (to be discussed). To have optical amplification, the medium has to be kept in a state of population inversion, i.e., in a state in which the number of atoms in the upper energy level is greater than that in the lower energy level-this is achieved by means of the pump.
2. Pumping source. The pump enables us to obtain such a state of population inversion between a pair of energy levels of the atomic system. When we have a state of population inversion, the input light beam can get amplified by stimulated emission.


Mirror
100\% reflecting

Semi-transparent Mirror
$\sim 90 \%$ reflecting
3. Optical resonator. A medium with population inversion is capable of amplification; however, for it to act as an oscillator, a part of the output energy must be fed back into the system. Such feedback is brought about by placing the active medium in a resonator; the resonator could be just a pair of mirrors facing each other.


Although Einstein proposed the theory of stimulated emission in 1917, the concept of population inversion to amplify the light beam came much-much later. According to Charles Townes,

The laser invention happened because I wanted very much to be able to make an oscillator at frequencies as high as the infrared in order to extend the field of microwave spectroscopy in which I was working. I had tried several ideas, but none worked very well. At the time I was also chairman of a committee for the navy that was examining ways to obtain very short-wave oscillators. In 1951, on the morning before the last meeting of this committee in Washington, I woke up early worrying over our lack of success. I got dressed and stepped outside to Franklin Park, where I sat on a bench admiring the azaleas and mulling over our problem.

Why couldn't we think of something that would work at high frequencies? I went through the possibilities, including, of course, molecules, which oscillate at high frequencies. Although I had considered molecules before, I had dismissed them because of certain laws of thermodynamics. But suddenly I recognized, "Hey, molecules don't have to obey such a law if they are not in equilibrium." And I immediately took a piece of paper out of my pocket and wrote equations to see if selection of excited molecules by molecular beam methods could produce enough molecules to provide a feedback oscillator. Wow! It looked possible.

I went back to my hotel and told Art Schawlow about the idea, since he was staying at the same place. . . . Its extension to waves as short as light came a few years later, after much excitement over the maser and as a result of my continued collaboration with Schawlow, then at Bell Labs. An essential element in this discovery, I believe, was my experience in both engineering and physics: I knew both quantum mechanics and the workings and importance of feedback oscillators.

## Population Inversion:

In the previous section we assumed that the atom is capable of interacting with radiation of a particular frequency $\omega$. However, if one observes the spectrum of the radiation due to spontaneous emissions from a collection of atoms, one finds that the radiation is not monochromatic but is spread over a certain frequency range. This would imply that energy levels have widths and atoms can interact over a range of frequencies. As an example, in Fig. below, it is shown that the $2 P$ level of hydrogen atom has a certain width $\Delta E(=\hbar \Delta \omega)$ so that the atom can absorb/emit radiation over a range of frequencies $\Delta \omega$. For the $2 P \rightarrow 1 S$ transition

$$
\Delta E \approx 4 \times 10^{-7} \mathrm{eV} \quad \Rightarrow \quad \Delta \omega \approx 6 \times 10^{-8} \mathrm{~s}^{-1}
$$

Since $\omega_{0} \approx 1.55 \times 10^{16} \mathrm{~s}^{-1}$, we get

$$
\frac{\Delta \omega}{\omega_{0}} \approx 4 \times 10^{-8}
$$



$$
\Delta E \approx 4 \times 10^{-7} \mathrm{eV}
$$

Thus, in general, $\Delta \omega \ll \omega_{0}$, showing the spectral purity of the source. We introduce the normalized line shape function $g(\omega)$ such that,

- Number of spontaneous emissions per unit time per unit volume with emitted frequency lying between $\omega$ and $\omega+d \omega=N_{2} A_{21} g(\omega) d \omega$
- Number of stimulated emissions per unit time per unit volume with emitted frequency lying between $\omega$ and $\omega+d \omega=N_{2} B_{21} u(\omega) g(\omega) d \omega$
- Number of stimulated absorptions per unit time per unit volume with absorbed frequency lying between $\omega$ and $\omega+d \omega=N_{1} B_{12} u(\omega) g(\omega) d \omega$,

Obviously,

$$
\int_{0}^{\infty} g(\omega) d \omega=1
$$

Thus the total number of stimulated emissions per unit time per unit volume is given by

$$
W_{21}=N_{2} \int_{0}^{\infty} B_{21} u(\omega) g(\omega) d \omega=N_{2} \frac{\pi^{2} c^{3}}{\hbar \tau_{s p} p_{0}^{3}} \int_{0}^{\infty} \frac{u(\omega)}{\omega^{3}} g(\omega) d \omega
$$

where we have used Eqs. (27) and (20). Now, for a near monochromatic radiation field (as is indeed the case for the laser), $g(\omega)$ is very sharply peaked at a particular value of $\omega\left(s a y, \omega^{\prime}\right)$,

$$
W_{21} \approx N_{2} \frac{\pi^{2} c^{3}}{\hbar \tau_{s p} n_{0}^{3}} \frac{g\left(\omega^{\prime}\right)}{\omega^{\prime 3}} U \quad\left[\frac{\mathrm{~A}_{21}}{\mathrm{~B}_{21}}=\frac{\hbar \omega^{3} n_{0}^{3}}{\pi^{2} c^{3}} \quad \& \quad \tau_{s p}=\frac{1}{A_{21}}\right]
$$

where $g\left(\omega^{\prime}\right)$ represents the value of the line shape function evaluated at the radiation frequency $\omega^{\prime}$ and $U$ represents the energy density associated with the radiation field.

$$
U=\int_{0}^{\infty} u(\omega) d \omega
$$

Now the energy density $U$ and the intensity $I_{\omega}$ are related through the following equation

$$
I_{\omega}=v U=\frac{c}{n_{0}} U
$$

where $v\left(=c / n_{0}\right)$ represents the velocity of the radiation field in the medium, $n_{0}$ being its refractive index. (The quantity $I_{\omega}$ represents energy per unit area per unit time) Thus the total number of stimulated emissions per unit time per unit volume is given by

$$
\begin{equation*}
W_{21}=N_{2} \frac{\pi^{2} c^{2}}{\hbar t_{\mathrm{sp}} n_{0}^{2}} \frac{g(\omega)}{\omega^{3}} I_{\omega} \tag{32}
\end{equation*}
$$

where we have dropped the prime on $\omega$. Similarly, the number of stimulated absorptions per unit time per unit volume is given by

$$
\begin{equation*}
W_{21}=N_{1} \frac{\pi^{2} c^{2}}{\hbar t_{\mathrm{sp}} n_{0}^{2}} \frac{g(\omega)}{\omega^{3}} I_{\omega} \tag{3}
\end{equation*}
$$

We next consider a collection of atoms and let a near monochromatic beam of frequency $\omega$ be propagating through it along the $z$ direction. To obtain an expression for the rate of change of the intensity of the beam as it propagates, we consider two planes of area $S$ perpendicular to the $z$ direction at $z$ and $z+d z$. The volume of the medium between planes $P_{1}$ and $P_{2}$ is $S d z$, and hence the number of stimulated absorptions per unit time is $W_{12} S d z$. Since each photon has an energy $\hbar \omega$, the energy absorbed per unit time in the volume element Sdz is

$$
\mathrm{W}_{12} \hbar \omega \mathrm{Sdz}
$$

Similarly, the corresponding energy gain (because of stimulated emissions) is

$$
\mathrm{W}_{21} \hbar \omega \mathrm{Sdz}
$$


where we have neglected the radiation arising out of spontaneous emissions, because such radiation propagates in random directions and is, in general, lost from the beam. Thus, the net amount of energy absorbed per unit time in the volume element $S d z$ is

$$
\left(\mathrm{W}_{12}-\mathrm{W}_{21}\right) \hbar \omega \mathrm{Sdz}
$$

If $I_{\omega}(z)$ represents the intensity of the beam per unit area per unit time, in plane $P_{1}$, then the total energy entering the volume element $S d z$ per unit time is
$I_{\omega}(z) S$
Similarly, if $I_{\omega}(z+d z)$ represents the intensity in plane $P_{2}$, then the total energy leaving the volume element per unit time is

$$
I_{\omega}(z+d z) S=I_{\omega}(z) S+\frac{\partial I_{\omega}}{\partial z} d z S
$$

Hence the net amount of energy leaving the volume element per unit time is

$$
I_{\omega}(z+d z) S-I_{\omega}(z) S=\frac{\partial I_{\omega}}{\partial z} d z S
$$

This must be equal to the negative of the energy absorbed by the medium between $z$ and $z+d z$. Thus,

$$
\frac{\partial I_{\omega}}{\partial z} S d z=-\left(W_{12}-W_{21}\right) \hbar \omega S d z=-\frac{\pi^{2} c^{2}}{\hbar t_{s p} \omega^{3} n_{0}^{2}} g(\omega) I_{\omega} \hbar \omega S d z\left(N_{1}-N_{2}\right)
$$

$$
\begin{align*}
& \frac{1}{I_{\omega}} \frac{\partial I_{\omega}}{\partial z}=\gamma  \tag{34}\\
& \gamma=\frac{\pi^{2} c^{2}}{\omega^{2} t_{s p} n_{0}^{2}}\left(N_{2}-N_{1}\right) g(\omega) \tag{35}
\end{align*}
$$

Since the line shape function $g(\omega)$ is very sharply peaked, the function $\gamma$ is also sharply peaked. Equation (34) can be readily integrated to give

$$
\begin{equation*}
I_{\omega}(z)=I_{\omega}(0) e^{\gamma z} \tag{36}
\end{equation*}
$$

Thus if $N_{1}>N_{2}$, then $\gamma$ is negative and the intensity of the beam decreases exponentially with $z$, with the intensity decreasing to $1 /$ e of its value at $z=0$ in a distance $1 / \gamma$. Hence at thermal equilibrium, since the number of atoms in the lower level is greater than that in the upper level, the intensity of the beam (as it propagates through the medium) decreases exponentially.

On the other hand, if there are more atoms in the excited level $\left(\mathrm{N}_{2}>\mathrm{N}_{1}\right)$ than in the lower level (i.e., there is a population inversion), then $\gamma>0$ and there will be an exponential increase in intensity of the beam; this is known as light amplification.

