Propositions

- A *proposition* is a statement or sentence that can be determined to be either true or false (but no both).
- Examples:
  - The only positive integers that divide 7 are 1 and 7 itself.
  - Buy two tickets for Friday concert.
  - Earth is the only planet in the universe that contains life.

Example

- Use variable to represent propositions
- P: 1+1=3
- P: It is raining outside
- P: Today is Tuesday

Connectives

If p and q are propositions, new *compound* propositions can be formed by using *connectives*

- Most common connectives:
  - Conjunction $\wedge$
  - Disjunction $\vee$
  - Negation $\sim$
  - Exclusive-OR $\underline{\vee}$
  - Condition $\rightarrow$
  - Bi-Condition $\leftrightarrow$

Example

- P: It is raining
- Q: It is cold

- Form a new compound statement by combining these two statements
- $P \wedge Q$: It is raining *and* it is cold
- $P \vee Q$: It is raining *or* it is cold

Truth table of conjunction

- The truth values of compound propositions can be described by *truth tables.*
- Truth table of *conjunction*

P	**Q**	**P $\wedge$ Q**
T	T	T
T	F	F
F	T	F
F	F	F

- P$\wedge$ Q is true only when both Pp and Q are true.

Example

- Let P = "A decade is 10 years"
- Let Q = "A millennium is 100 years"
- $P \wedge Q$ = "A decade is 10 years" and "A millennium is 100 years"
- If P is true and Q is false then conjunction is false

Truth table of disjunction

- The truth table of *disjunction* is

P	Q	P ∨ Q
T	T	T
T	F	T
F	T	T
F	F	F

- $p \vee q$ is false only when both p and q are false
  - Example: p = "John is a programmer", q = "Mary is a lawyer"
  - p v q = "John is a programmer or Mary is a lawyer"

Example

- Let P = “A decade is 10 years”
- Let Q = “A millennium is 100 years”
- $P \wedge Q$ = “A decade is 10 years” and “A millennium is 100 years”
- If P is true and Q is false then conjunction is false

Negation

- Negation of P: in symbols ~P

P	~P
T	F
F	T

- ~P is false when P is true, ~P is true when P is false
  - Example, P : "John is a programmer"
  - ~P = "John is not a programmer"

- E.g

- P: Paris is the capital of England
- ~P: Paris is not capital of England

Logical equivalence

- Two propositions are said to be *logically equivalent* if their truth tables are identical.

P	Q	$\sim P \vee Q$	$P \to Q$
T	T	T	T
T	F	F	F
F	T	T	T
F	F	T	T

- Example: $\sim P \vee Q$ is *logically equivalent* to $P \to Q$

Converse

- The *converse* of $p \rightarrow q$ is $q \rightarrow p$

p	**q**	**$p \rightarrow q$**	**$q \rightarrow p$**
T	T	T	T
T	F	F	T
F	T	T	F
F	F	T	T

These two propositions
are <u>not</u> logically equivalent