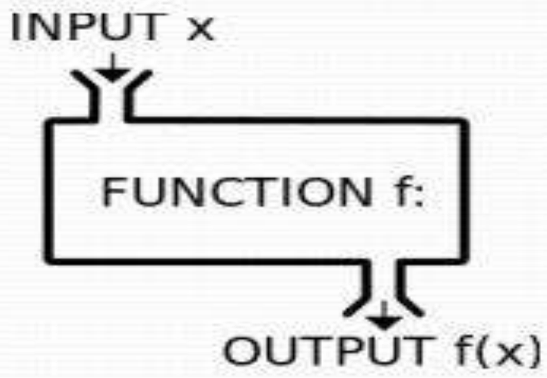
Functions

Definition: let A and B be nonempty sets, a **function** f from A to B, denoted by $f : A \rightarrow B$, is an assignment of each element of A to **exactly one** element of B.

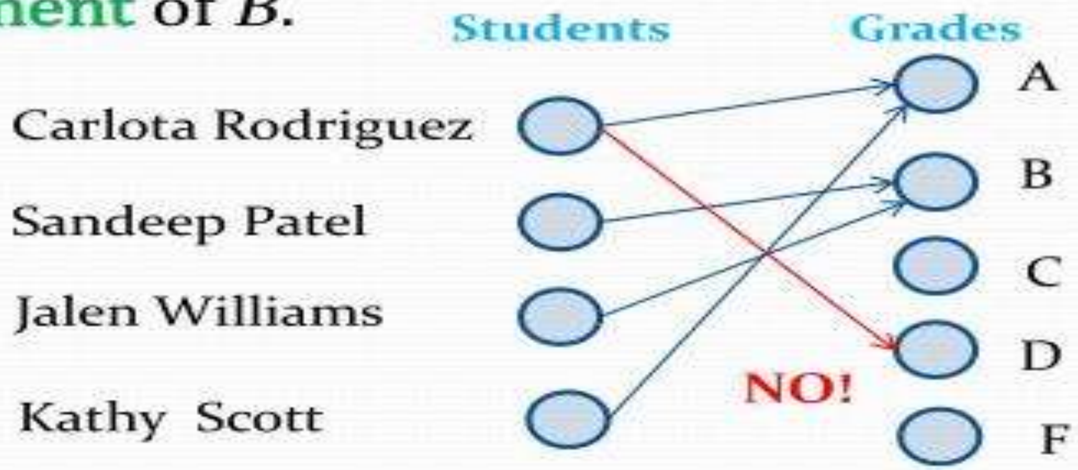
- We write $f(a) = b$ if b is the **unique element** of B assigned by the function f to the element a of A.

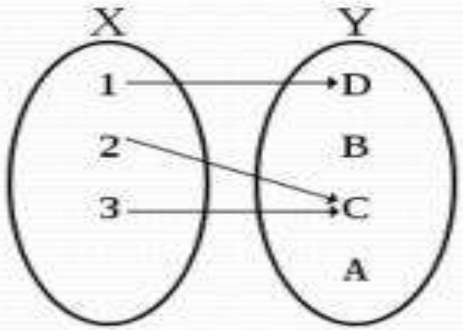
- Functions are sometimes called **mappings** or **transformations**.



Functions

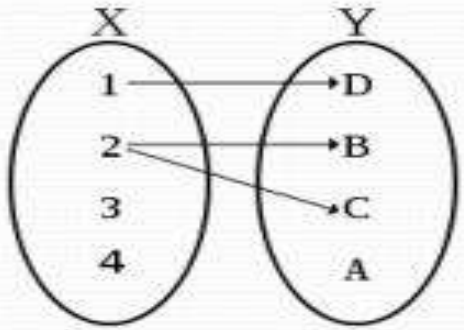
Example: $f : A \rightarrow B$, is an assignment of each element of A to **exactly one element** of B.





YES!

NO!



Functions

- A function $f : A \to B$ can also be defined as a subset of $A \times B$ (a relation). This subset is restricted to be a relation where no two elements of the relation have the same first element.

- Specifically, a function f from A to B contains one and only one ordered pair (a, b) for every element $a \in A$; first element.

$$\forall x[x \in A \to \exists y[y \in B \wedge (x, y) \in f]]$$

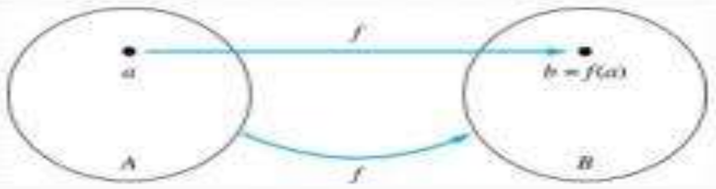
and

$$\forall x, y_1, y_2[[(x, y_1) \wedge (x, y_2)] \in f \to y_1 = y_2]$$

Functions

Given a function $f : A \to B$. We say f *maps* A to B,

- A is called the **domain** of f,
- B is called the **codomain** of f.



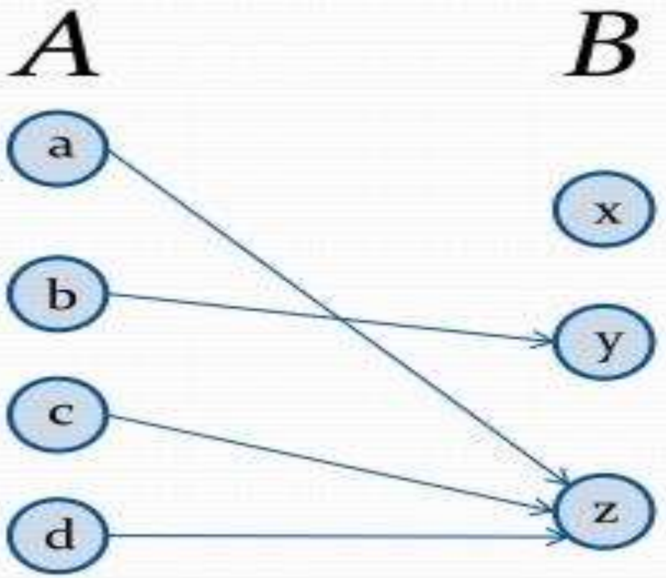
- If $f(a) = b$,
  - then b is called the **image** of a under f,
  - a is called the **preimage** of b.
  - "$f(a)$" is also known as the **range**.
- Two functions are **equal** when they have the same domain, the same codomain and map each element of the domain to the same element of the codomain.

Representing Functions

- Functions may be specified in different ways:
  1. An explicit statement of the assignment.
     - For instance, students and grades example.
  2. A formula:
     - $f(x) = x + 1$
  3. A computer program:
     - A Java program that when given an integer n, produces the n^{th} Fibonacci Number (covered in the next section and also in Chapter 5).

Questions

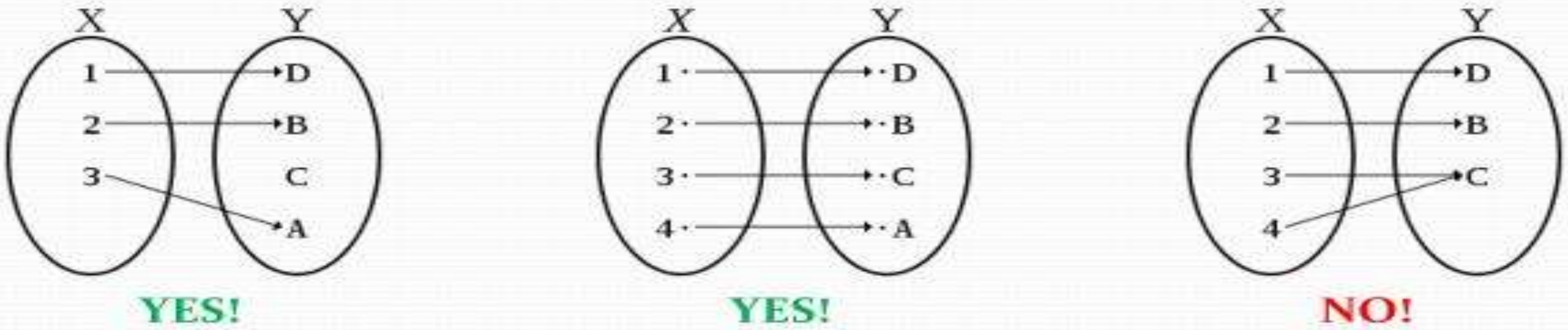
$f(a) = ?$	**z**
The image of d is ?	**z**
The domain of f is ?	***A***
The codomain of f is ?	***B***
The preimage of y is ?	**b**
The preimage(s) of z is (are) ?	**{a, c, d}**



Injections

Definition: a function f is said to be **one-to-one** if and only if for all **a** and **b** in the domain, $f(a) = f(b)$ implies that $a = b$.

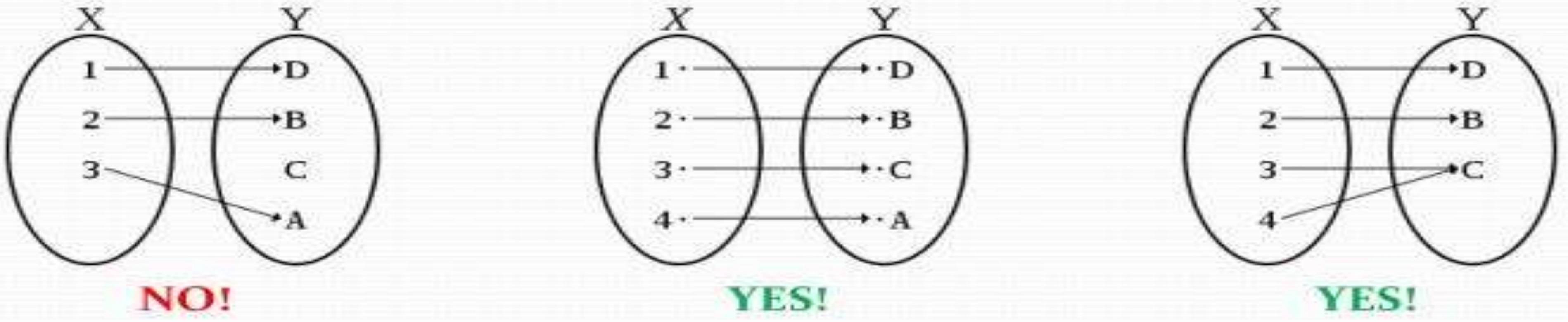
- A function is said to be an **injection** if it is one-to-one.



Surjections

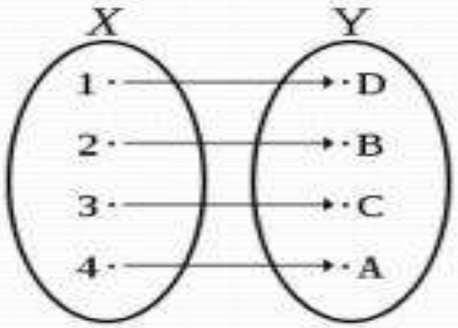
Definition: a function *f* from *A* to *B* is called **onto**, if and only if for every element b □ Y, there is an element a □ X with $f(a) = b$.

- A function *f* is called a **surjection** if it is onto.

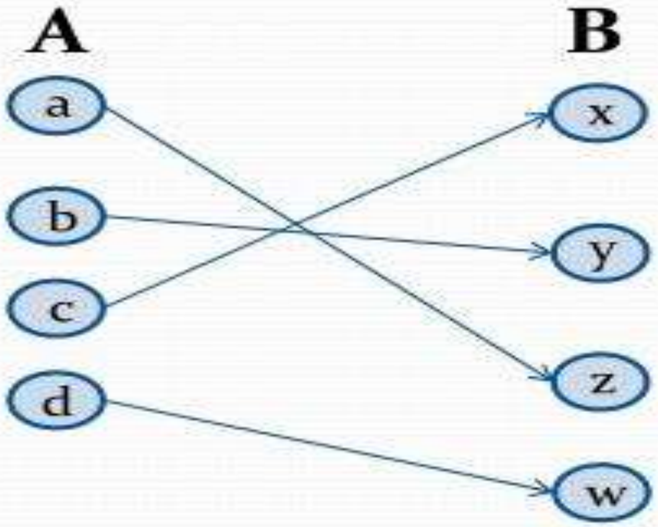


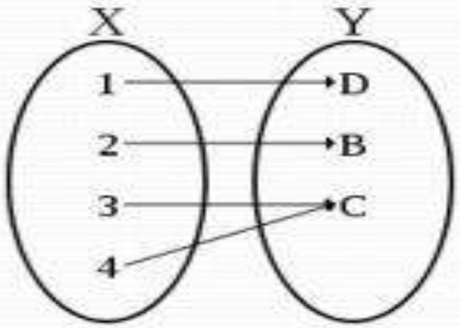
Bijections

Definition: a function *f* is a **bijection** *(one-to-one correspondence)*, if it is both one-to-one and onto, i.e., both surjective and injective.



YES!





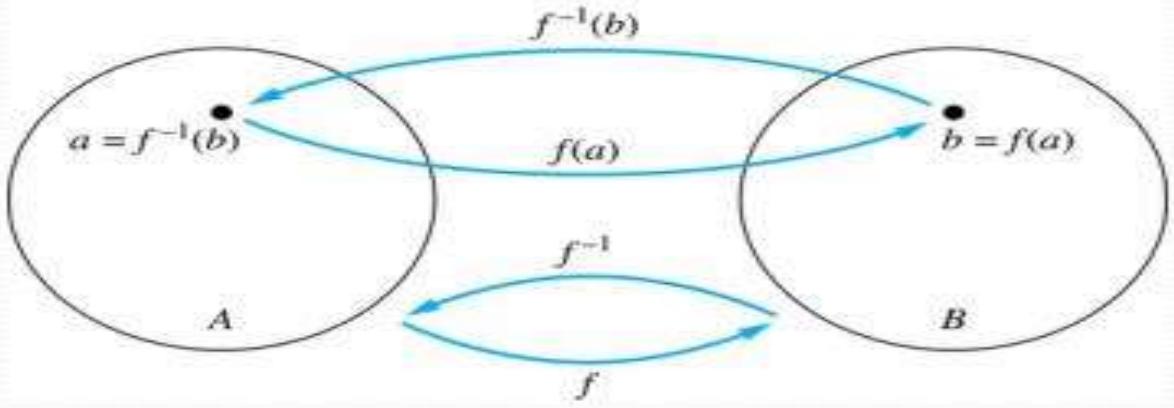
NO!
surjection but not injection

Inverse Functions

Definition: let f be a bijection from A to B. Then the **inverse of** f, denoted by f^{-1}, is the function from B to A defined as

$$f^{-1}(y) = x \text{ iff } f(x) = y$$

- No inverse exists unless f is a bijection.



Inverse Functions